GOAL
Find and describe patterns and use inductive reasoning

VOCABULARY
A conjecture is an unproven statement that is based on observations.
Inductive reasoning is a process that involves looking for patterns and making conjectures.
A counterexample is an example that shows a conjecture is false.

EXAMPLE 1
Describing a Visual Pattern
Sketch the next figure in the pattern.

\[
\begin{array}{c}
\text{•} \\
\text{•} \\
\text{•} \\
\end{array}
\]

Solution
Each figure looks like the one before it except that it has rotated $90^\circ$. The next figure will have the smaller circle in the lower-left quarter of the bigger circle.

Exercise for Example 1
1. Sketch the next figure in the pattern.

\[
\begin{array}{c}
\text{△} \\
\text{△} \\
\text{△} \\
\end{array}
\]
**Example 2**

**Describing a Number Pattern**

Describe a pattern in the sequence of numbers. Predict the next number.

a. 5, 3, 1, −1, . . .

b. 1, −4, 9, −16, . . .

c. \(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\)

**Solution**

a. These are consecutive odd numbers, but listed backwards starting with 5. The next number is −3.

b. These numbers look like consecutive perfect squares, except that every other one is negative. The next number is 25.

c. Each number is \(\frac{3}{2}\) times the previous number. The next number is \(\frac{1}{10}\).

**Exercises for Example 2**

Describe a pattern in the sequence of numbers. Predict the next number.

2. 1, 2, 6, 24, . . .

3. 0, 3, 8, 15, 24, . . .

**Example 3**

**Making a Conjecture**

Complete the conjecture.

**Conjecture:** The product of two consecutive even integers is divisible by ?.

**Solution**

List some specific examples and look for a pattern.

**Examples:**

\[
\begin{align*}
2 \times 4 &= 8 = 8 \times 1 & 6 \times 8 &= 48 = 8 \times 6 & 10 \times 12 &= 120 = 8 \times 15 \\
4 \times 6 &= 24 = 8 \times 3 & 8 \times 10 &= 80 = 8 \times 10 & 12 \times 14 &= 168 = 8 \times 21
\end{align*}
\]

**Conjecture:** The product of two consecutive even integers is divisible by 8.
Practice with Examples

For use with pages 3–9

Exercises for Example 3

Complete the conjecture based on the pattern you observe in the specific cases.

4. Conjecture: For any two numbers \( a \) and \( b \), the product of \( (a + b) \) and \( (a - b) \) is always equal to \( ? \).
   \[
   (2 + 1) \times (2 - 1) = 3 = 2^2 - 1^2 \\
   (3 + 2) \times (3 - 2) = 5 = 3^2 - 2^2 \\
   (4 + 2) \times (4 - 2) = 12 = 4^2 - 2^2 \\
   (6 + 3) \times (6 - 3) = 27 = 6^2 - 3^2
   \]

Finding a Counterexample

Show the conjecture is false by finding a counterexample.

Conjecture: All odd numbers are prime.

Solution

The conjecture is false. Here is a counterexample: The number 9 is odd and is a composite number, not a prime number.

Exercise for Example 4

Show the conjecture is false by finding a counterexample.

5. The square of the sum of two numbers is equal to the sum of the squares of the two numbers. That is, \((a + b)^2 = a^2 + b^2\).